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Economy and efficiency of swimming at the surface with fins of different size and stiffness

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Abstract The aim of this study was to investigate how fins with varying physical characteristics affect the energy cost and the efficiency of aquatic locomotion. Experiments were performed on ten college swimmers who were asked to swim the dolphin kick while using a monofin (MF) and to swim the front crawl kick with a small-flexible fin (SF), a large-stiff fin (LS) and without fins (BF, barefoot). The energy expended to cover one unit distance (C) was highest for BF ($C = 10.6 \pm 1.8 \text{ kJ m}^{-1} \text{ kg}^{-1}$ at 0.8 m s^{-1}) and decreased by about 50% with LS, 55% with SF and 60% with MF, allowing for an increase in speed (for a given metabolic power) of about 0.4 m s^{-1} for MF and of about 0.2 m s^{-1} for SF and LS (compared with BF). At any given speed, the fins for which C was lower were those with the lowest kick frequency (KF): $\text{KF} = 1.6 \pm 0.22 \text{ Hz}$ at 0.8 m s^{-1} (for BF) and decreased by about 40% for SF, 50% for LS and 60% for MF. The decrease in KF from BF to SF–LS and MF was essentially due to the increasing surface area of the fin which, in turn, was associated with a higher Froude efficiency (η_F). η_F was calculated by computing the speed of the bending waves moving along

the body in a caudal direction (as proposed for the undulating movements of slender fish): it increased from 0.62 ± 0.01 in BF to 0.66 ± 0.03 in SF and 0.67 ± 0.04 in LS reaching the highest values (0.76 ± 0.05) with MF. No single fin characteristic can predict a swimmer's performance, rather the better fin (i.e. MF) is the one that is able to reduce most KF at any given speed and hence to produce the greatest distance per kick ($d = v/\text{KF}$). The latter indeed increased from $0.50 \pm 0.01 \text{ m}$ in BF to about $0.90 \pm 0.05 \text{ m}$ in SF and LS and reached values of $1.22 \pm 0.01 \text{ m}$ in MF.

Keywords Fin swimming · Propelling efficiency
Kick frequency · Energy balance

Introduction

Commercially available fins come in a variety of designs and materials and require different kicking styles. Fins differ mainly in their size (length, width and surface area), mass, density and stiffness. A fin's stiffness depends not only on the material used (rubber or fibre-glass) but also on the presence of flanges and vents which also affect the fluid-dynamics of the water flow along the fin during the stroke. All these “construction factors” could affect the economy and the efficiency of fin swimming in different ways. In general terms, existing data on the energetics of fin swimming show that large and rigid fins are energetically demanding but improve the maximal thrust per kick, whereas flexible, small-sized fins improve the economy of swimming at sub-maximal “cruising” speeds (Pendergast et al. 1996).

The economy of fin swimming (the energy expenditure to cover one unit distance, C) can be calculated from the ratio \dot{E}/v , where \dot{E} is the net metabolic power and v is the average speed of progression. At constant and sub-maximal speed $\dot{E} = \dot{V}\text{O}_2$: therefore C can be assessed by simple measures of oxygen consumption. Whereas several papers report data of $\dot{V}\text{O}_2$ or C in fin

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swimming (e.g. Donald and Davidson 1954; Goff et al. 1956; Morrison 1973; Pendergast et al. 1996, 2003a, b; Specht et al. 1957; Zamparo et al. 2002), data on the efficiency of this form of locomotion are not well documented. The overall efficiency of locomotion is defined as the ratio of total mechanical power to metabolic power ($\eta_O = \dot{W}_{tot}/\dot{E}$); hence, to calculate η_O , the mechanical power must be quantified; this is not a simple task in aquatic locomotion.

The concepts of efficiency and economy are not interchangeable: economy needs efficiency but efficiency does not imply economy. As pointed out by Minetti (2004), an efficient locomotion is one where most of the metabolic power is transformed into mechanical power, but it is possible that some of this mechanical power is not necessary for propulsion, resulting in a worse economy. If the mechanical power output is close to the minimum necessary and most of it contribute to progression, locomotion is also economical.

As indicated by Cavagna and Kaneko (1977), the total mechanical power of locomotion (\dot{W}_{tot}) is the sum of two terms: the power needed to accelerate and decelerate the limbs with respect to the centre of mass (the internal power, \dot{W}_{int}) and the power needed to overcome external forces (the external power, \dot{W}_{ext}). In aquatic locomotion, the latter can be further partitioned into \dot{W}_d , the power to overcome drag that contributes to useful thrust and \dot{W}_k , the power that does not contribute to thrust. Both types of power give water kinetic energy but only \dot{W}_d effectively contributes to propulsion (Alexander 1977, 1983; Daniel 1991; Daniel et al. 1992).

In fin swimming, these three components of \dot{W}_{tot} can be separately assessed (Zamparo et al. 2002) thus allowing the computation of the Froude ($\eta_F = \dot{W}_d/\dot{W}_{ext}$), the propelling ($\eta_P = \dot{W}_d/\dot{W}_{tot}$), the hydraulic ($\eta_H = \dot{W}_{ext}/\dot{W}_{tot}$), the performance ($\eta_D = \dot{W}_d/\dot{E}$) and the overall ($\eta_O = \dot{W}_{tot}/\dot{E}$) efficiencies. A detailed definition of these terms appears in Fig. 1 and in Materials and methods section).

The effect on the economy of swimming due to the use of a small, flexible fin (Apollo Biofin pro) was investigated in a previous study (Zamparo et al. 2002) in

which we observed a reduction of C in comparison to swimming the leg kick without fins. The improvement in the energy cost of locomotion was attributed to the concurrent reduction of the internal (\dot{W}_{int}) and kinetic (\dot{W}_k) work rates and was correlated to a significant increase in η_F , η_P , η_D and η_H in comparison with swimming without fins.

Fins with larger propelling surfaces (e.g. when using a monofin) are expected to lead to higher η_F and η_P in comparison with swimming without fins or smaller surface fins. However, the effects of other fin characteristics (such as their buoyancy or stiffness) on the energetics and biomechanics of fin swimming are more difficult to predict.

The aim of this study was to investigate how fins with varying physical characteristics affect the energy cost of aquatic locomotion and the biomechanical factors that determine it. This may also allow the determination of which fin design characteristics optimizes this form of human locomotion in water.

Materials and methods

The subjects of the study were ten college swimmers who were members of a Division I University men's swimming team (University of Buffalo, NY, USA). Their average (\pm SD) body mass was 75.5 ± 8.7 kg, their average stature was 1.80 ± 0.04 m and their average age was 19.7 ± 1.8 years.

They were asked to swim the flutter (front crawl) kick without fins (BF, barefoot), with a small and flexible fin (SF, Apollo Biofin Pro) and with a large and stiff fin (LS, Mares Attack). They were also asked to swim the dolphin kick while using a Monofin (MF, Finis). The physical characteristics of the investigated fins are reported in Table 1.

Underwater weight of the fins was measured by means of a load cell (AEP transducers, I) while the fins were completely submerged in order to calculate their density.

Flexural rigidity (EI , $N\ m^2$) was calculated as described in detail by Pendergast et al. (2003a) from the

Fig. 1 A flow diagram of the steps of energy conversion in aquatic locomotion (adapted from Daniel 1991). See text for details

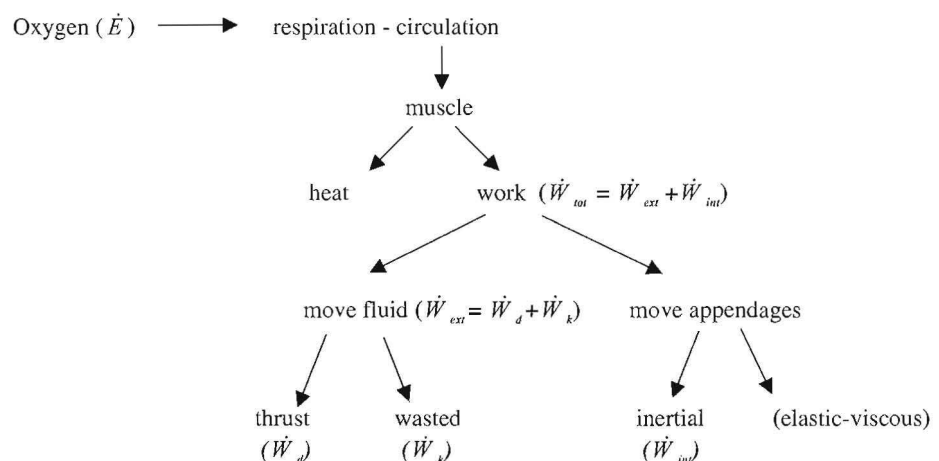
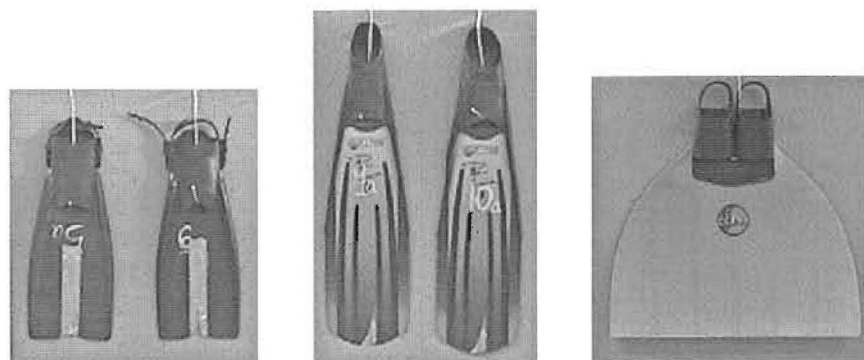


Table 1 Physical characteristics of the fins

	<i>SF #5</i>	<i>SF #6</i>	<i>LS #9</i>	<i>LS #10</i>	<i>MF</i>
blade SA (m²)	0.08	0.08	0.12	0.12	0.30
blade length (cm)	33	33	57	57	42
blade width (*) (cm)	22	22	22	22	70
fin mass (kg)	1.39	1.32	1.18	1.28	1.30
fin density (gr · cm³)	1.033	1.074	1.299	1.269	1.457
flexural rigidity (N · m²)	1.26	1.26	5.45	5.45	4.24

Footnote: SF: Apollo Biofin Pro; LS: Mares Attack; MF: Monofin Finis; SA: surface area; * at the trailing edge.

deflection (δ , m) of the fin's blade when adding weights (P , N) to the trailing edge at a distance L (m) from the fulcrum: $EI = PL^3/(3\delta)$ (Gere and Timoshenko 1990).

The fin's surface area was obtained from digital photographs (Cybershot DSC - F505, Sony, Japan), using automatic pixel counting of a manually selected outlines and metric unit conversion using a reference grid of known size (NIH Image 1.62, USA).

Experimental procedure

The subjects swam at the water surface in an annular pool 2.5 m wide, 2.5 m deep and of 60 m circumference over the swimmer's path. The swimmers were paced by a platform moving at constant speed about 60 cm above the water surface. The speed of the swimmer was set by means of an impeller type flow meter (PT - 301, MEAD Inst. Corp., Riverdale, NY, USA) placed 1.5 m in front of the swimmer and connected to a tachometer (F1-12 P Portable indicator, MEAD Inst. Corp., NY, USA). The subjects were requested to swim with the arms hyper-extended over the head and the thumbs joined with the palms facing down (hence, forward propulsion was obtained by kicking the legs only).

The experiments were carried out over a range of speeds that could be accomplished aerobically, corresponding to 0.7, 0.8, 0.9 m s⁻¹ for BF, 0.7, 0.8, 0.9, 1.0 m s⁻¹ for SF and LS and 0.8, 0.9, 1.0, 1.1 m s⁻¹ for MF (one trial per subject and per speed).

Energy cost (economy) of swimming

In all conditions (BF, SF, LS and MF) the subjects were requested to complete an incremental swimming test at the above indicated speeds. After 3 min of steady-state swimming at each speed, the expired gas was collected (for about 60 s) into an aerostatic balloon through a water proof inspiratory and expiratory valve and hose system supported by the platform. The $\dot{V}O_2$ values were determined by means of the standard open circuit method: the gas volume was determined using a dry gas meter (Harvard dry gas meter, USA) and the O₂ and CO₂ fractions in the expired air were determined by means of a previously calibrated mass spectrometer (MGA 1100, Perkin Elmer, CA, USA). Net $\dot{V}O_2$ (above rest, assumed to be 5 ml min⁻¹ kg⁻¹) was converted to W assuming that 1 mlO₂ consumed by the human body yields 20.9 J (which is strictly true for a respiratory quotient of 0.96) and divided by the speed v to yield the energy cost of swimming per unit of distance (C) in kJ m⁻¹. During the metabolic data collection the kick frequency (KF, Hz) was also recorded.

Kinematic analysis

During the experiments video records were taken with a sampling rate of 50 Hz (Handy Cam Vision, Sony, Japan) while the subjects passed in front of an underwater window. Black tape markers were applied on

selected anatomical landmarks in order to facilitate the following video analysis. The camera was placed in a fixed position at a distance of 3 m from the plane of motion with its focal plane perpendicular to it; the movements on the left side of the subject were recorded. After the experiments, the data were downloaded to a PC and digitized using a commercial software package (Peak Motus, CO, USA). A calibration factor (for each subject, speed and condition) was obtained by comparing the distance between the left hip (great trochanter) and the left knee (lateral epicondyle) in pixel with the anatomical distance in meters (measured and recorded for each subject and in each experimental session). The calibration factor was calculated in correspondence of a frame in which the leg was fully extended. The errors due to the small camera distance or to slight deviations of the movement out of the plane of motion (e.g. caused by body roll) were rather small (about 2–3%); e.g. a calculated tight distance of 0.437 ± 0.011 m (average over 75 frames) compared to an anatomical distance of 0.452 m.

Trunk inclination (TI)

Trunk inclination was measured from the angle between the shoulder (acromion process) and the hip (great trochanter) segment and the horizontal. Trunk inclination varied sinusoidally within a cycle; the data reported in this paper are the average values over one cycle

Kick depth (KD)

Kick depth was measured as the maximal difference in the vertical position of the two ankles (lateral malleolous).

Froude efficiency (η_F)

During steady-state aquatic locomotion, the Froude efficiency of the undulatory movement of a slender fish is given by: $\eta_F = (c + v)/2c$, where c is the wave speed and v is the progression speed (e.g. Lighthill 1975; Daniel 1991). Waves of bending similar to the ones described for slender fish were reported for subjects swimming the butterfly stroke (Ungerechts 1983; Sanders et al. 1995) and the leg kick (Zamparo et al. 2002) and this formula was utilized to calculate the Froude efficiency of the leg kick in swimming with and without fins (Zamparo et al. 2002). From the above equation it is evident that the closer the swimming and the backward wave velocities, in absolute terms, the higher the Froude efficiency. Such a progression would correspond to sliding, while undulating, with very little displacement of water.

The velocity of the backward wave (c) can be calculated on the basis on the 2D coordinates of the hip, knee and ankle joints: each coordinate reach its minimum/maximum displacement with a phase shift represented

by the time lag. The distance between the anatomical landmarks (e.g. the thigh or shank length) divided by the corresponding time lag between the waves minima/maxima gives the velocity of the wave along the body (Ungerechts 1983; Zamparo et al. 2002).

Kinetic work (W_k)

Froude efficiency is also defined as: $\eta_F = W_d/(W_d + W_k)$ (e.g. Lighthill 1975; Daniel 1991) and it is maximum when W_k is minimum (i.e. when little water is uselessly accelerated). Thus, the term W_k can be calculated from data of η_F (calculated as described above) and data of active body drag (W_d , calculated as described below) as follows: $W_k = (W_d/\eta_F) - W_d$.

Internal work (W_{int})

The internal work of the leg kick was computed from video analysis. The location of nine anatomical landmarks (wrist, elbow, shoulder, neck, hip, knee, ankle, heel, toe tip) was digitized over one complete swimming cycle in two subjects for LS and in one subject for MF. On the assumption that bilateral swimming movements are symmetrical, the 2D coordinates obtained from the body side proximal to the camera were duplicated (shifted by half a cycle for LS) and the swimming cycle was reconstructed for the whole body. From the obtained coordinates and from standard anthropometric tables (Dempster et al. 1959), the position and the linear and angular speed of each body segment were calculated, from which the position of the body centre of mass was also derived. The extra mass of the fins was taken into account in order to compute the segment mass/total mass fraction of each body segment (it was assumed that most of the extra mass of the fin was added to the centre of mass of the foot). The sum of the increases, over the time course, of the absolute rotational kinetic energy and of the relative (with respect to the body centre of mass) linear kinetic energy of adjacent segments over one cycle were then computed by a custom software package (Minetti 1998) in order to calculate W_{int} .

As indicated by Zamparo et al. (2002), the internal work rate of the leg kick could be described by an equation of the form $\dot{W}_{int} = k (2KD)^2 \times KF^3$ where the term k is related to the inertia parameters of the moving body segments, KD is the kick depth and KF is the kick frequency. This model equation was utilized to estimate k for LS and MF by means of a multiple non-linear regression (Systat 5, USA) on the basis of the experimentally determined values of \dot{W}_{int} , KD and KF.

The internal work rate was then calculated for each subject, speed and condition on the basis of the experimentally determined values of KD and KF and of the appropriate value of k .

The values of the constant k were previously calculated for the leg kick without fins ($k_{BF} = 13.93$, $n = 10$,

$r^2=0.976$) and for the leg kick with fins (the same as the SF fins utilized in this study) ($k_{SF}=25.55$, $n=10$, $r^2=0.832$) by Zamparo et al. (2002).

Active body drag (\dot{W}_d)

Active body drag was measured as described by di Prampero et al. (1974) for MF and LS only. Known masses (from 0.5 to 4 kg) were attached to the swimmer's waist by means of a rope and a safety belt that did not interfere with the swimming mechanics. The rope passed through a system of pulleys fixed to the monitoring platform in front of the swimmer, thus allowing the force to act horizontally along the direction of movement. This force (the added drag, D_a) leads to a reduction of the swimmer's active body drag (D_b) and, at constant speed, is associated with a consequent reduction of $\dot{V}O_2$: the energy required to overcome D_b becomes zero when D_a and D_b are equal and opposite. At the beginning of the experimental session a load was applied to the pulley system and the subject was asked to attain the requested speed. After 3 min, once the steady state was attained, the oxygen consumption was determined as described above. After 1 min the expired gas collection was terminated and the load on the pulley was diminished by about 0.5 kg. This procedure was repeated until, in the last step, the subject swam freely (without any added load). The swimmer's D_b was estimated, at any given speed, by extrapolating the $\dot{V}O_2$ versus D_a relationship to resting $\dot{V}O_2$. The power dissipated against drag was then calculated from the product of the active body drag times the speed ($\dot{W}_d = D_b v$).

The active body drag was calculated in a previous study (Zamparo et al. 2002) for subjects swimming without (BF) or with fins (the same as the SF fins utilized in this study). Since the anthropometric characteristics of the two groups of subjects are similar (three out of ten subjects participated to both studies) we felt it fair to assume that the relationship between \dot{W}_d and speed previously determined was also applicable to this group of subjects. The values of \dot{W}_d for BF and SF reported in this paper are therefore those calculated and reported previously (Zamparo et al. 2002).

The propelling, hydraulic, performance and overall efficiencies

The efficiency with which the overall mechanical power (\dot{W}_{tot}) produced by the swimmer is transformed into useful propulsion (useful mechanical power \dot{W}_d) is termed propelling efficiency and is given by ($\eta_P = \dot{W}_d / \dot{W}_{tot}$).

The efficiency with which the overall mechanical power (\dot{W}_{tot}) produced by the swimmer is transformed into external power ($\dot{W}_{ext} = \dot{W}_k + \dot{W}_d$) is termed hydraulic efficiency and is given by ($\eta_H = \dot{W}_{ext} / \dot{W}_{tot}$).

Hence, since the Froude efficiency is defined as ($\eta_F = \dot{W}_d / \dot{W}_{ext}$), it follows that $\eta_P = \eta_F \times \eta_H$ and hence, that $\eta_P = \eta_F$ if the internal power is nil or negligible (and if the hydraulic efficiency is close to 1). On the other hand, the propelling efficiency will be lower than the Froude efficiency the higher the internal power and the lower the hydraulic efficiency.

Whereas the Froude, propelling and hydraulic efficiencies refer to the mechanical partitioning only, the performance and the overall efficiency take into account also the metabolic expenditure.

The efficiency with which the metabolic power input (\dot{E}) is transformed into useful propulsion (useful mechanical power output, \dot{W}_d) is termed performance efficiency and is given by ($\eta_D = \dot{W}_d / \dot{E}$).

The efficiency with which the metabolic power input (\dot{E}) is transformed into mechanical power output (\dot{W}_{tot}) is termed overall efficiency and is given by $\eta_O = \dot{W}_{tot} / \dot{E}$. It follows that $\eta_O = \eta_D / \eta_P$.

Statistics

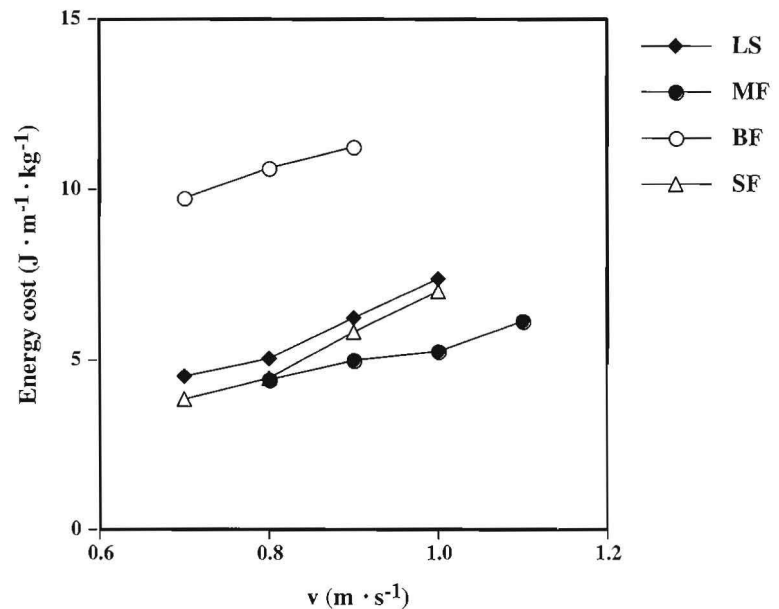
Correlation between $\dot{V}O_2$ and D_a for each condition was calculated by the least squares method. The differences in the measured variables (e.g. C , D_b , \dot{W}_{int} ...) as determined while kicking the legs without (BF) or with different fins (SL, LF and MF) were compared by the paired Student's t test ($n=10$), with the level of significance set at ≤ 0.05 . The differences in the values of D_b as determined in this and in the previous study (Zamparo et al. 2002) were compared by means of an unpaired Student's t test. The comparisons were made at the speeds of 0.7, 0.8 and 0.9 m s⁻¹ when comparing BF with SL and LS and at the speeds of 0.8 and 0.9 m s⁻¹ when comparing BF with MF.

Results

The energy cost of swimming the leg kick (C , kJ m⁻¹) in the four conditions (BF, SF, LS and MF) is reported in Fig. 2 as a function of the speed (v , m s⁻¹). The energy expended to cover one unit distance was highest for the flutter kick without fins (BF); it decreased by about 50% with the use of large and stiff fins (LS) and by about 55% when small and flexible fins were used (SF) ($P < 0.001$, at all speeds and for both conditions). Swimming the dolphin kick with the monofin (MF) required even less energy in comparison with the flutter kick (about 60% at comparable speeds, $P < 0.001$, at both speeds) and allowed for an increase in speed, for a given metabolic power, of about 0.4 m s⁻¹ compared to swimming the flutter kick with legs alone (about 0.2 m s⁻¹ for SF and LS).

The values of η_F for the four conditions are reported in Fig. 3 as a function of the speed (v , m s⁻¹): η_F increased significantly from 0.62 ± 0.01 in BF to 0.66 ± 0.03 in SF and 0.67 ± 0.04 in LS reaching the

Fig. 2 Energy cost (C , $\text{J m}^{-1} \text{kg}^{-1}$) of swimming the flutter kick as a function of speed (v , m s^{-1}) measured without fins (BF barefoot), with small and flexible fins (SF) and large and stiff fins (LS). The energy cost of swimming the dolphin kick with a monofin (MF) is also reported as a function of speed



highest values (0.76 ± 0.05) with the monofin (MF, $P < 0.02$, at all speeds and for all conditions). Swimming with the monofin was the only case for which η_F showed a tendency to increase with the speed. The increase in η_F from BF to MF, means that less energy is “wasted” to give water kinetic energy; indeed the values of \dot{W}_k (for a given \dot{W}_d) decreased (albeit not significantly) from BF to SF and LS and were significantly reduced in comparison to BF when swimming with the monofin ($P < 0.001$, at both speeds).

The average values (± 1 SD) of kick frequency, kick depth, trunk inclination and active body drag are reported in Table 2. Fins only slightly decreased the kick depth in respect to the leg kick: by 4% for LS, 14% for LS and 18% for MF (the difference is significant only for MF: $P < 0.02$, at both speeds); but caused a large reduction in the kick frequency. This reduction was

highest for fins with the larger surface area: about 45% for SF, 50% for LS and 60% for MF ($P < 0.001$, at all speeds and for all conditions).

As indicated in Table 2, the average angle of the trunk with the horizontal decreased with increasing speed in BF, SF and LS ($\text{TI} = 12.5 \times v - 22.6$, $r^2 = 0.745$, $n = 11$) whereas it was fairly stable in MF. Trunk inclination was slightly increased by the use of fins (of about 10% for SF, LS and MF) in comparison with swimming the leg kick without fins, the differences, however, were not significant.

No significant differences were also observed, at comparable speeds, in the active body drag (D_b) among the different conditions (Table 3).

The values of k for LS and MF turned out to be: $k_{\text{LS}} = 44.6$ ($n = 10$, $r^2 = 0.959$) and $k_{\text{MF}} = 24.0$ ($n = 4$, $r^2 = 0.953$). The high coefficients of determination sug-

Fig. 3 Froude efficiency (η_F) while swimming the flutter kick as a function of speed (v , m s^{-1}) measured without fins (BF), with small and flexible fins (SF) and large and stiff fins (LS). The Froude efficiency of swimming the dolphin kick with a MF is also reported as a function of speed

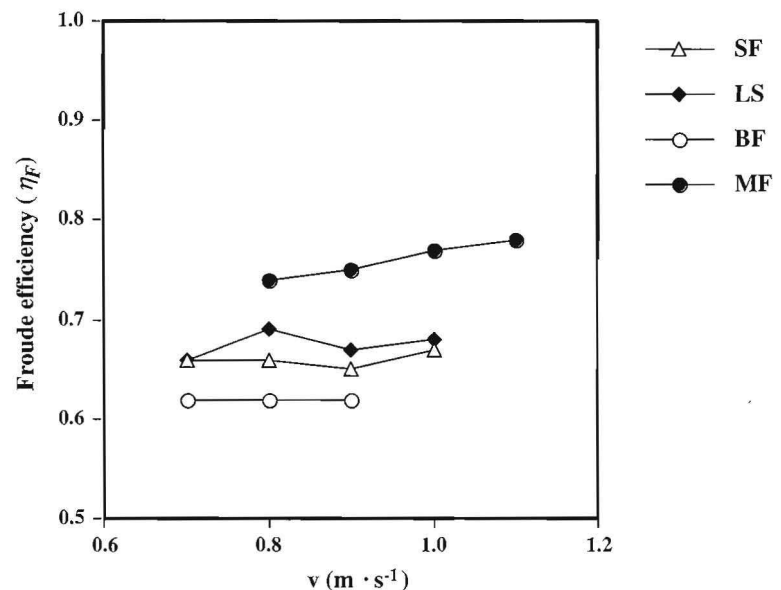


Table 2 Average values (± 1 SD) of kick frequency (KF), kick depth (KD) active body drag (D_b) and trunk inclination (TI) as measured when swimming at the indicated speeds (v) with three different kinds of fins (SF, LS, MF) or without fins (BF)

	v (m s ⁻¹)	KF (Hz)	D_b (N)	KD (m)	TI (deg)
BF	0.7	1.43 \pm 0.22	23.6 \pm 5.8 ^a	0.38 \pm 0.06	-2.8 \pm 4.8
	0.8	1.60 \pm 0.22	29.5 \pm 4.6 ^a	0.38 \pm 0.07	-0.9 \pm 4.0
	0.9	1.78 \pm 0.17	38.9 \pm 8.8 ^a	0.37 \pm 0.06	-1.2 \pm 3.0
SF	0.7	0.80 \pm 0.24	28.3 \pm 10.8 ^a	0.31 \pm 0.05	-4.1 \pm 4.7
	0.8	0.89 \pm 0.22	22.9 \pm 5.4 ^a	0.34 \pm 0.05	-3.2 \pm 4.7
	0.9	1.04 \pm 0.22	41.8 \pm 13.5 ^a	0.33 \pm 0.05	-2.0 \pm 5.3
LS	1.0	1.22 \pm 0.24	41.9 \pm 10.3 ^a	0.33 \pm 0.06	-0.4 \pm 4.1
	0.7	0.71 \pm 0.21	26.1 \pm 4.3	0.37 \pm 0.13	-4.5 \pm 5.1
	0.8	0.83 \pm 0.18	35.2 \pm 5.2	0.35 \pm 0.10	-3.4 \pm 6.5
MF	0.9	0.96 \pm 0.24	38.5 \pm 5.1	0.37 \pm 0.10	-1.8 \pm 5.0
	1.0	1.11 \pm 0.26	46.5 \pm 6.7	0.37 \pm 0.07	-2.2 \pm 3.0
	0.8	0.65 \pm 0.09	33.6 \pm 10.9	0.30 \pm 0.07	-1.8 \pm 7.0
	0.9	0.74 \pm 0.18	40.1 \pm 7.4	0.32 \pm 0.04	-2.9 \pm 4.6
	1.0	0.82 \pm 0.15	46.5 \pm 9.1	0.31 \pm 0.02	-1.7 \pm 3.9
	1.1	0.92 \pm 0.27	54.1 \pm 5.1	0.31 \pm 0.05	-1.8 \pm 4.5

BF leg kick without fins, SF Apollo Biofin (small and flexible), LS Mares Attack (large and stiff), MF Monofin^aData from Zamparo et al. (2002)

Table 3 Average values (± 1 SD) of net metabolic expenditure (\dot{E}), of the power needed to overcome frictional forces (\dot{W}_d), to impart kinetic energy to the water (\dot{W}_k) and to overcome inertial forces (\dot{W}_{int}) along with the total mechanical power (\dot{W}_{tot}) as measured when swimming at the indicated speeds (v) with three different kinds of fins (SF, LS, MF) or without fins (BF)

	v (m s ⁻¹)	\dot{E} (W)	\dot{W}_d (W)	\dot{W}_k (W)	\dot{W}_{int} (W)	\dot{W}_{tot} (W)
BF	0.7	516 \pm 78	16.5 \pm 4.0 ^a	10.2 \pm 0.2	25.8 \pm 10.9	52.4 \pm 11.0
	0.8	641 \pm 110	23.6 \pm 3.7 ^a	14.7 \pm 1.2	36.3 \pm 16.3	74.7 \pm 16.2
	0.9	765 \pm 85	35.0 \pm 7.9 ^a	21.5 \pm 1.1	45.5 \pm 14.9	102.0 \pm 15.0
SF	0.7	203 \pm 45	19.8 \pm 7.6 ^a	10.2 \pm 2.1	5.8 \pm 6.0	35.9 \pm 7.5
	0.8	269 \pm 42	20.6 \pm 4.8 ^a	10.5 \pm 1.4	8.6 \pm 4.3	39.7 \pm 4.9
	0.9	395 \pm 80	37.6 \pm 12.1 ^a	20.1 \pm 2.0	12.0 \pm 5.4	69.7 \pm 6.7
LS	1.0	529 \pm 77	41.7 \pm 10.3 ^a	21.0 \pm 2.3	19.6 \pm 6.4	82.3 \pm 8.3
	0.7	239 \pm 55	18.2 \pm 3.0	9.5 \pm 2.3	8.1 \pm 3.9	35.8 \pm 5.9
	0.8	305 \pm 65	28.2 \pm 4.1	13.0 \pm 3.9	12.0 \pm 4.7	53.1 \pm 8.6
MF	0.9	423 \pm 78	34.7 \pm 4.6	16.9 \pm 3.1	22.2 \pm 15.9	73.8 \pm 17.1
	1.0	557 \pm 57	46.5 \pm 6.7	22.3 \pm 3.5	33.1 \pm 13.6	101.9 \pm 15.0
	0.8	267 \pm 39	26.9 \pm 8.7	9.6 \pm 3.1	2.5 \pm 1.1	39.0 \pm 11.8
	0.9	338 \pm 67	36.1 \pm 6.7	12.0 \pm 4.2	4.1 \pm 2.7	52.2 \pm 9.0
	1.0	397 \pm 50	46.5 \pm 9.1	14.4 \pm 5.3	5.2 \pm 2.4	66.1 \pm 12.6
	1.1	508 \pm 57	59.5 \pm 5.6	16.8 \pm 4.4	6.7 \pm 4.9	83.0 \pm 7.7

BF leg kick without fins, SF Apollo Biofin (small and flexible), LS Mares Attack (large and stiff), MF Monofin^aData from Zamparo et al. (2002)

Table 4 Average values (± 1 SD, of all subjects and at all speeds) of overall (η_O), propelling (η_P), hydraulic (η_H), Froude (η_F) and performance (η_D) efficiency when swimming with three kinds of fins (SF, LS, MF) or without fins (BF). The parameters reported in this table are related as follows: $\eta_O \times \eta_P = \eta_D$ and $\eta_P = \eta_F \times \eta_H$

	(η_O)	(η_P)	(η_H)	(η_F)	(η_D)
BF	0.12 \pm 0.02	0.34 \pm 0.07	0.55 \pm 0.12	0.62 \pm 0.01	0.04 \pm 0.01
SF	0.17 \pm 0.04	0.54 \pm 0.06	0.81 \pm 0.08	0.66 \pm 0.03	0.09 \pm 0.02
LS	0.17 \pm 0.04	0.50 \pm 0.08	0.74 \pm 0.10	0.68 \pm 0.04	0.09 \pm 0.02
MF	0.16 \pm 0.03	0.70 \pm 0.06	0.92 \pm 0.04	0.76 \pm 0.05	0.11 \pm 0.02

BF leg kick without fins, SF Apollo Biofin (small and flexible), LS Mares Attack (large and stiff), MF Monofin

gest that \dot{W}_{int} can be accurately estimated from the experimentally determined values of KD and KF and from the estimated values of k . This was done also for BF and SF, for which $k_{BF} = 13.93$ ($n = 10$, $r^2 = 0.976$) and $k_{SF} = 25.55$ ($n = 10$, $r^2 = 0.832$), as reported by Zamparo et al. (2002).

The so obtained values of internal work rate (\dot{W}_{int} , W) as a function of the speed (v , m s⁻¹) for all conditions (BF, SF, LS and MF) are reported in Table 3, along with the data of \dot{W}_d , \dot{W}_k , \dot{W}_{tot} and \dot{E} . This Table shows that the internal work rate for the monofin (MF) is rather small, as it can be expected on theoretical

grounds and that \dot{W}_{int} is largely reduced when fins are used. In comparison with the leg kick (BF) a 60, 75 and 90% reduction in \dot{W}_{int} is observed for LS, SF and MF, respectively ($P < 0.01$, at all speeds and in all conditions). The decrease of \dot{W}_{int} and \dot{W}_k brought about by the use of fins leads to a reduction of the total mechanical power output (in comparison with BF) of about 20, 35 and 50% for LS, SF and MF, respectively ($P < 0.005$, at all speeds and in all conditions).

Finally, in Table 4 are reported the average values (at all speeds) of all types of efficiencies for aquatic locomotion. η_H , η_P , η_O and η_D increased significantly from BF to SF/LS reaching the highest values with MF ($P < 0.01$, at all speeds and for all conditions).

Discussion

From the definitions of overall ($\eta_O = \dot{W}_{\text{tot}}/\dot{E}$) and propelling ($\eta_P = \dot{W}_d/\dot{W}_{\text{tot}}$) efficiency it is apparent that the metabolic power in aquatic locomotion is given by

$$\dot{E} = (\dot{W}_d/\eta_P) \eta_O^{-1} \quad (1)$$

For any given speed, and by knowing that $C = \dot{E}/v$ and $\dot{W}_d = D_b \cdot v$:

$$C = (D_b/\eta_P) \eta_O^{-1} \quad (2)$$

Equation 2 indicates that differences in the energy cost of swimming, at any given speed, have to be attributed to differences in η_P and/or η_O or to changes in hydrodynamic resistance. Data reported in this study show that differences in D_b are negligible among conditions (see discussion below) and that the differences in η_O were rather small. Hence the most important parameter affecting the economy of swimming the leg kick with different kinds of fins is the propelling efficiency: the larger η_P the lower C (e.g. the higher the economy of locomotion).

Propelling efficiency is given by the product of hydraulic and Froude efficiency ($\eta_P = \eta_H \times \eta_F$). Data reported in this paper indicate that the observed changes of C due to the use of fins have to be attributed to differences of both η_H and η_F .

Differences in hydraulic efficiency among conditions result from differences of internal work rate ($\eta_H = 1$ if \dot{W}_{int} is nil or negligible, a condition for which $\eta_P = \eta_F$). Data reported in Table 2 indicate that no major differences in KD are found among conditions and hence, that the internal work rate ($\dot{W}_{\text{int}} = k(2KD)^2 \times \text{KF}^3$) is essentially dependent on the frequency of the kick: the lower KF the lower \dot{W}_{int} and the higher η_H (and η_P).

The kick frequency is a parameter which largely affects the Froude efficiency too (KF is the reciprocal of the time period of the wave speed). As shown by several authors, with increasing speed the wave period decreases (and hence KF increases) whereas the wavelength remains essentially constant (e.g. Sanders et al. 1995; Webb 1971; Zamparo et al. 2002). Thus a reduction of

KF leads to a decrease of the wave speed and hence to an increase in η_F (and η_P), *ceteris paribus*.

The fact that KF decreases from BF to SF–LS and MF is essentially due to the increasing surface area of the fin (see Table 1). Indeed, as indicated by Alexander (1977), the efficiency of a propeller is higher if a large mass of fluid is accelerated to a low velocity than if a small mass of water is accelerated to a high velocity. Hence, the larger the surface area of the fin, the lower the speed of movement (the lower the kick frequency) and higher the propelling efficiency.

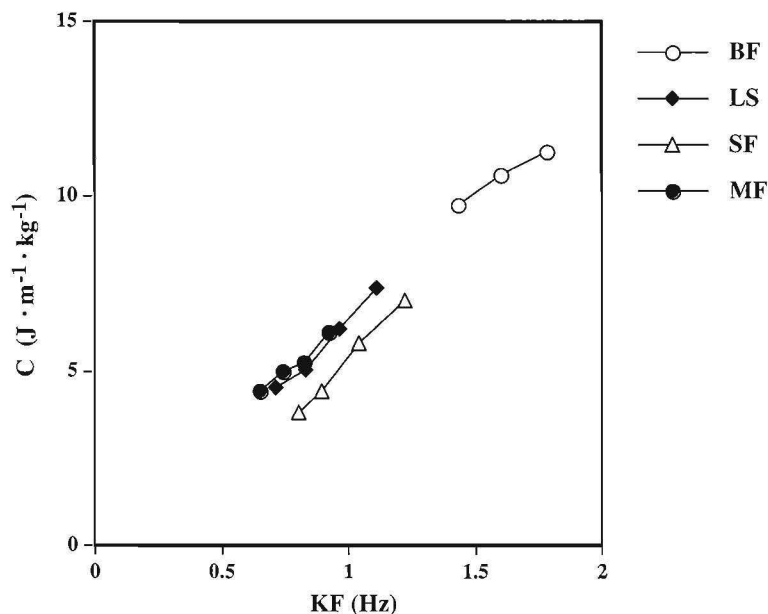
Thus, a decrease of KF is expected to increase η_F , decrease η_H and hence increase η_P (and decrease C). That this is indeed the case is indicated in Fig. 4 where the data of KF and C are reported for all conditions and at all speeds. This figure shows that all types of fins reduce the energy cost of locomotion in comparison to BF by decreasing the frequency of the kick. The decrease of C is larger for SF (at any given KF) in comparison to LS due to the lower value of k which further reduces \dot{W}_{int} . Indeed, k_{LS} was found to be twice the value of k_{SF} , the difference being related to differences in buoyancy/density rather than to differences in mass, which was essentially the same (Table 1).

The most efficient (with the highest η_P) and economical (with the lowest C) fin, however, is the one that is able to reduce most KF and hence to produce the greatest distance per kick ($d = v/\text{KF}$). As shown by Table 4 and Fig. 5, the distance per kick is about twice that of BF (0.50 ± 0.01 m) for SF (0.87 ± 0.03 m) and LS (0.94 ± 0.04 m) and reaches values of 1.22 ± 0.01 m in MF. Figure 5 moreover shows (1) that the differences in d are strictly related to differences in η_P (and hence of C); (2) that large and “heavy” fins (LS) are characterized by approximately the same propelling efficiency of fins with smaller surface but better buoyancy (SF); and (3) that swimming with a single fin operated simultaneously by the two lower limbs (MF) allows for the largest reduction in KF (and KD) in respect to swimming the flutter kick (even with two fins of comparable surface area, such as LS), and it is therefore associated with the largest distance per stroke, the better propelling efficiency and thus to the higher economy and cruising speed (see Table 2 and Fig. 2).

Propelling efficiency is almost unaffected by the speed for BF, SF and LS and show a tendency to increase with increasing speed for MF; it is therefore fair to assume that the observed differences in efficiency (and hence of economy) would be maintained at the higher speeds that can be attained during competitions (e.g. about 2 m s^{-1} over the 1,000 m distance for MF, as reported by Minetti 2004) and that the advantage of using a monofin would, if any, increase with increasing speed.

The increase of η_P observed when swimming with MF (from 0.69 at 0.8 m s^{-1} to 0.72 at 1.1 m s^{-1}) is consistent with the finding that the energy cost of swimming increases as a function of the speed less steeply than for the other fins. Indeed, as calculated by Minetti (2004) on

Fig. 4 Energy cost (C , $\text{J m}^{-1} \text{kg}^{-1}$) as a function of the kick frequency (KF, Hz) in the four conditions (BF, SF, LS, MF)



the basis of the word records of monofin swimming, the energy expended to cover a unit distance with a monofin is almost speed independent in a wide range of “aerobic” speeds (e.g. for races lasting more than 5–10 min).

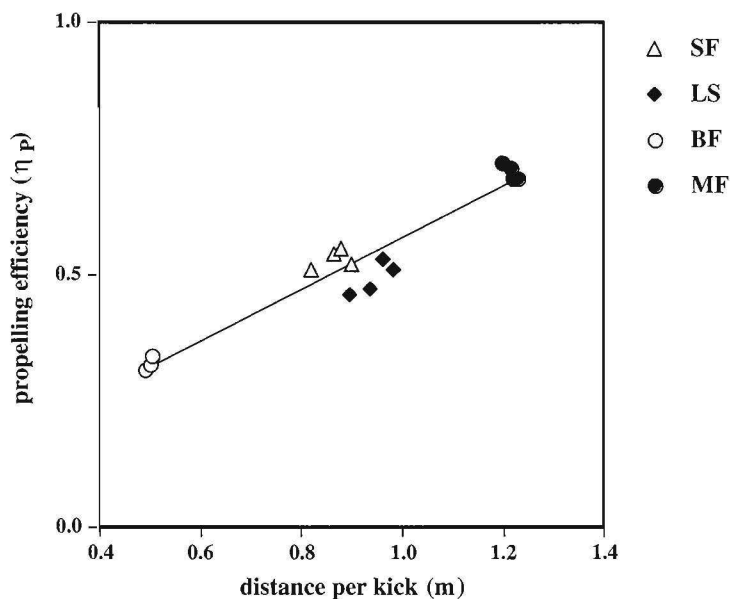
Active body drag and the work estimates

Active body drag was found to be the same when using different fins or swimming barefoot at the surface (Table 2). This could be due to the fact that we did not measure D_b in the BF and SF conditions but we utilized data collected on a similar but different group of subjects (Zamparo et al. 2002). However, in the previous

paper we have shown that no significant differences in D_b are detectable when the same subjects swim with or without small, flexible fins and in this paper we have shown that no differences are detectable when the same subjects swim with large, rigid fins or with the monofin. These data, taken together, indicate indeed that large differences in the fin's characteristics (stiffness, surface area, density and length) have small or no effect on hydrodynamic resistance.

This conclusion is supported by the fact that, at the speeds investigated in this study, total drag is determined mostly by pressure drag (Mollendorf et al. 2004). Pressure drag depends on the frontal area of the swimmer which is proportional to the trunk inclination which was

Fig. 5 Propelling efficiency (η_p) as a function of the distance covered per kick ($d_k = v/KF$) in the four conditions (BF, SF, LS, MF). The relationship between the two variables is well described by $\eta_p = 0.514 + 0.059 \times d_k$, $n = 15$, $r^2 = 0.926$, $P < 0.001$



not significantly different among conditions (Table 2). In addition, as water flow is transitional over most of the body and it becomes turbulent below the knee (Mollendorf et al. 2004) any "difference" below this area would indeed be expected to have a small effect on hydrodynamic resistance.

The method utilized in this study to determine the external mechanical work associated to hydrodynamic resistance (W_d) yields values that are larger than those reported in the literature and obtained by means of different methods (for a review see Wilson and Thorp 2003). The reader is referred to a recent paper for a detailed discussion of this method (Zamparo et al. 2005). An overestimation of W_d could lead to an overestimation of the efficiencies of aquatic locomotion but this does not invalidate the conclusions of this paper about the determinants of the differences in the efficiencies we observed among conditions.

The method utilized in this study to determine the internal mechanical work (W_{int}) is based on the assumption that W_{int} is dominated by the kinematic part of it, i.e. the work needed to reciprocally accelerate limbs with no movement of the body centre of mass. Actually, there has to be a (small) component of the internal work caused by overcoming the friction of connective tissue and other biological structures which slide or deform as the limb/trunk reciprocal position changes. Therefore, if any, the contribution of W_{int} to W_{tot} could have been slightly underestimated.

Finally, the external mechanical work associated to unusefully accelerating water backward (W_k) has been estimated from the bending wave speed. While this method relies on consolidated achievements in the physics of propulsion, the level of approximation needs to be appreciated. It is likely that computational fluid dynamics and particle velocimetry, by theoretically and experimentally analysing the fine hydrodynamics near to the swimming body, will be able to better refine the estimation of the induced/parasite drag as caused by vortices and flow separation.

A comparison with data of η_P reported in the literature

Movement analysis of fin swimming (or, more generally, of underwater undulatory movements) can be approached by means of two methods: the analysis of water displacement around the fin (e.g. Ungerechts et al. 1999; Arellano 1999) and the analysis of the movements of the swimmer and/or of the fin (e.g. Pendergast et al. 2003a; Zamparo et al. 2002).

Reflective particles, tufts, injected bubbles and/or dye can be utilized to visualize the flow behaviour around the swimmer (for a review see Arellano 1999). With these methods it can be shown that the undulatory movements of the breaststroke and flutter kick generate vortices (of different size, rotating speed and direction) in the wake behind the feet. Since η_P is larger if a large mass of water is accelerated to a small velocity than vice versa (Alex-

ander 1977, 1983), investigating the fluid momentum of the vortex rings can give insight on the propelling efficiency of swimming.

As described in studies of fish locomotion (e.g. Lauder and Drucker 2002) the vortex momentum can indeed be calculated and the associated thrust and lift forces can be mathematically derived. With these methods the "external forces" (the energy output into the fluid) rather than the energy input supplied by the musculature are investigated. This is an interesting and quite novel approach to the study of human locomotion in water, however, to our knowledge, no data of η_F and/or η_P as calculated with these methods are yet reported in the literature (at least in relation to human swimming).

The works focusing on the internal physiological mechanisms that govern fish locomotion are based on the observation that rhythmic locomotory patterns, arising from segmental muscle activity, generate the undulatory body bending which propels the fish forward (e.g. Rome et al. 1993). As indicated by Lighthill (1960), the mechanism by which these forces can be transmitted to the water can be investigated by comparing the water flow around the fish to that occurring over a thin and flexible wave plate, thus yielding theoretical estimates of thrust and η_F .

Recently, Pendergast et al. (2003a) applied the wave plate theory to the movements of the leading and trailing edge of fins in subjects swimming underwater (fin signature analysis) with eight different types of fins, among which were the SF and LS fins tested in this study. The values of Froude efficiency calculated by these authors are remarkably close to those reported in this study (0.60–0.70) even if our values were obtained when swimming at the surface (instead of underwater) and with a much simpler method (from measures of wave speed and progression speed).

The conclusions drawn by Pendergast et al. (2003a) in their fin's evaluation are in agreement with the results of our study: there is no single fin characteristic (stiffness, surface, splits, vents etc.) that can predict a diver's performance; rather, the best indicators of performance are kick depth and frequency which should be decreased as much as possible.

Propelling efficiency in swimmers has been estimated by Toussaint and coworkers by means of the MAD system: the values of η_P while swimming by using the arm stroke (with the legs floated by a pull buoy) were found to be of about 0.55 (e.g. Toussaint et al. 1988). Values of propelling efficiency for the front crawl were recently reported by Zamparo et al. (2005) in subjects swimming at the surface without ($\eta_P = 0.46$) or with ($\eta_P = 0.54$) fins (the same as the SF fins utilized in this study). These values are comparable to those reported in this study for the leg kick with fins (SF = 0.54 ± 0.06 and LS = 0.50 ± 0.08). The propelling efficiency of swimming by kicking the legs without fins is, on the other hand, much lower than when swimming the arm stroke/the front crawl (BF = 0.34 ± 0.07) whereas it reaches values comparable to those attained by fish (e.g. Webb 1971) when a monofin is utilized (MF = 0.70 ± 0.06). Propelling efficiency of

human powered propeller-driven boats can, on the other hand, be as high as 90% (Abbott et al. 1995). For a more detailed comparison of the methods to estimate η_P in aquatic locomotion the reader is referred to Pendergast et al. (2003c) and to Zamparo et al. (2002, 2005).

Fins as locomotory tools

As indicated in Table 4, the overall efficiency is essentially the same when swimming with different types of fins ($\eta_O = 0.16\text{--}0.17$) in spite of the differences in their physical characteristics, in the partial efficiencies and in the energy cost of locomotion.

Fins can be regarded as passive tools for enhancing aquatic locomotion (Zamparo et al. 2002; Minetti 2004); in analogy with other tools for locomotion on land they do not supply any additional energy to the body but provide compensation for limitations in the anatomical design (e.g. the small surface area of the feet) and act by limiting the internal and/or external mechanical work (by reducing W_k and W_{int} rather than W_d). As previously shown for locomotory tools on land (e.g. bicycles, Minetti et al. 2001) for a given motion/locomotion the so obtained decrease in W_{tot} allows for a proportional decrease in C so that overall efficiency ($\eta_O = W_{tot}/C$) is essentially unaffected by the type of "tool" utilized.

As discussed by Zamparo et al. (2002) and Minetti (2004), the gain in propulsion that can be obtained by swimming with fins is far from being commensurate with what muscles are expected to produce based on their performance on land locomotion. The overall efficiency of the leg kick either without (0.10) or with fins (0.16–17) is indeed much lower than the maximal theoretical value of 0.25–0.30 that can be obtained in isolated muscles working in aerobic conditions (Woledge et al. 1985).

Conclusions

The data presented in this study show that there is no single fin characteristic (stiffness, surface, splits, vents etc.) that can predict a swimmer's performance; rather, the best indicator is the kick frequency which should be decreased as much as possible to reduce the energy demands of this form of locomotion. Indeed, (1) the kick frequency is the major determinant of both the Froude and the hydraulic efficiency (the two factors determining the propelling efficiency of locomotion in water), and (2) changes of KF and of propelling efficiency are directly related to the differences in energy cost observed among different fins.

Factors which allow for an increase in the propelling efficiency are an increase in the fin's surface area and a decrease in the fin density: hence, large and "heavy" fins (LS) were found to be characterized by approximately the same economy and efficiency of fins with smaller surface but better buoyancy (SF). Swimming with a single fin operated simultaneously by the two lower

limbs (MF) is more efficient (the propelling efficiency is higher) than swimming the flutter kick with two fins of comparable surface area (LS) and allows for a much better economy, and thus to a higher cruising speed.

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